the omission was however a material one, inasmuch as this expression for the general co-efficient serves to connect my formulæ with Leverrier's development, *Annales de l'Obser. de Paris*, t. I. (1855) pp. 275–330 and 358–383, and I resume the question for the purpose of supplying it.

On the Proposition 38 *of the Third Book of Newton's Principia.* By I. Todhunter.

In this Proposition Newton undertakes to determine the figure of the Moon. The Moon is supposed to be a homogeneous fluid, drawn into an oblong shape by the disturbing action of the Earth; the rotation of the Moon on its axis, and the revolution of the Earth with the Moon round the Sun, are disregarded. The Proposition is connected by Newton with one that precedes, in which the Earth is supposed to be fluid, and to be similarly acted on by the Moon.

Let M denote the mass of the Earth, and m the mass of the Moon. Suppose that the Earth is a homogeneous fluid, and that, owing to the disturbing action of the Moon, it assumes the form of an ellipsoid of revolution round the major axis. Let B denote the minor axis, and B + H the major axis, where H is supposed small compared with B; let k denote the distance between the centres of the Earth and of the Moon. Then we shall have approximately

$$\mathrm{H} = \frac{15}{4} \cdot \frac{m}{\mathrm{M}} \cdot \frac{\mathrm{B}^4}{k^3}$$

Newton does not explicitly give this formula; but his numerical results in Propositions 36 and 37 appear to have been deduced from it; and it can be obtained readily in the same way as Newton employed elsewhere: see Whewell *on the Free Motion of Points*, third edition, page 202. The formula was obtained by D. Bernoulli in his prize essay on the Tides, Chapter IV., Article 8.

In like manner let b denote the minor axis, and $b + h$ the major axis of the ellipsoid of revolution, which the Moon becomes when disturbed by the Earth; then

$$h = \frac{15}{4} \cdot \frac{\mathrm{M}}{m} \cdot \frac{b^4}{k^3}$$

Hence by division we obtain

$$\frac{h}{\mathrm{H}} = \left(\frac{\mathrm{M}}{m}\right)^2 \left(\frac{b}{\mathrm{B}}\right)^4 \qquad (1)$$

Newton, however, in his Proposition 38, instead of (1), makes

a statement in words which is equivalent to the following formula :—

$$\frac{h}{\mathrm{H}} = \frac{\mathrm{M}}{m} \cdot \frac{b}{\mathrm{B}} \qquad (2)$$

Newton himself gives no reason for the statement.

In the *Astronomiæ Elementa*, by David Gregory, published at Oxford in 1702, a proposition is ennunciated, which is substantially equivalent to the formula (2), and is supported by a folio page of general reasoning, which is, however, altogether unsatisfactory ; see page 387 of that work.

In the Jesuits' edition of the *Principia* a note is given, which is founded on the process of David Gregory, and is also quite inadmissible.

Newton's error was pointed out with some emphasis by Paul Frisi ; see pages 174 and 186 of his *De Gravitate universali*, 1768, page 135 of the second volume of his *Cosmographia*, 1775, and page 170 of the third volume of his *Opera*, 1785.

If the error had been generally admitted, and the correction adopted, it would of course have been superfluous to recur to the matter ; but so far as I have been able to find, no attention has been given to it. Writers and commentators have sometimes touched on Propositions 36 and 37 in such a manner that we might naturally have expected an allusion to Proposition 38, if only by way of warning ; but nothing is said about it.

For a more decisive case, we may refer to *Robison's Mechanical Philosophy*, 1804. On page 512, he speaks of Frisi's *Cosmographia* as a very masterly performance on the subject of the figure of the planets ; and in his page 517, he alludes to Newton's Proposition III. 38, without any remark as to the criticism on it by Frisi.

But the most remarkable case is that furnished by the book of Dr. Whewell's, which I have already cited. On page 206, Newton's statement is reproduced without any indication whatever of its untenable character. This seems very strange, when we reflect that Dr. Whewell paid great attention to Newton's work, and also to the Theory of the Tides, with which the Proposition has some connexion.

Lord Brougham touches on the subject in his *Analytical View of Sir Isaac Newton's Principia;* he says on his page 295, "The tide in the Moon caused by our Earth is to the tide in our sea caused by the Earth, as the mass of the Earth to the mass of the Moon." I presume that on the second occasion where the word *Earth* occurs, it is a misprint for *Moon ;* and then the statement amounts to the formula,

$$\frac{h}{\mathrm{H}} = \frac{\mathrm{M}}{m}$$

This agrees neither with (1) nor with (2), and is of course quite wrong.

Laplace has considered the problem of the figure of the Moon in his *Théorie du Mouvement et de la figure elliptique des Planètes*. He takes into account the rotation of the Moon on its axis; but otherwise his solution is consistent with Frisi's result, and not with Newton's; see page 115 of the work. He does not mention either Newton or Frisi.

The Proposition to which this note relates is of no practical importance; but even in pure theory it is well to be accurate and consistent: I have therefore thought it would be useful to draw attention to a curious error which has indeed been corrected, but certainly not obliterated.

The Source of the Solar Heat. By Maxwell Hall, B.A.

Let us suppose that the mass of the Sun is slowly, but continually, contracting; then, in consequence of the enormous mass subjected to this contraction, an enormous amount of heat will be developed, and it will be found that the rate or amount of contraction necessary to produce the amount of heat radiated by the Sun into space, is so remarkably small, that ages must elapse before the effect of this contraction can become visible to us at our comparatively great distance from the Sun.

In order to show that this is the case, let one foot and one second be taken as the units of space and time, and suppose that each unit of volume of the Sun's mass contracts by the same amount in the same time, so that if z_0 be the linear contraction of the Sun's radius r_0 in one second, and if z be the contraction of any other length r, measured from the centre, and for the same duration of time, then $\frac{z}{z_0} = \frac{r}{r_0}$. The effect of this contraction may thus be compared to a series of intermittent pulsations, acting throughout the whole of the mass, and tending to diminish the volume. Let g_0 be the force of gravity at the surface of the Sun, and let g be the force of gravity at any point within the Sun's mass considered homogeneous, whose distance is r from the centre; then $\frac{g}{g_0} = \frac{r}{r_0}$. Again, let ϱ be the mean density of the Sun's mass, so that the weight of any thin concentric shell, whose radius is r and thickness ∂r, will be $4\pi g \varrho r^2 \partial r$; and, since every unit of mass in this shell falls through z feet towards the centre in a second of time, $4\pi g \varrho z r^2 \partial r$ will be the kinetic energy generated and destroyed every second by this shell alone; and therefore $\int_0^{r_0} 4\pi g z r^2 d r$ will be the whole kinetic energy destroyed every second of time, and we proceed to find the corresponding amount of heat evolved.